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NON-MODULAR ELLIPTIC CURVES AS CALCULATE SOLUTIONS FOR PROBLEMS OF P.FERMAT, G.FREY, A.POINCARE AND A.BEAL

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Abstract

The mail hoal of the article is to consider to common solution the system equations of P.Fermat, G.Fray and A.Beal and its application to the non-modular elliptic curves.
It is exact proof to folloving facts:

1. **Hypothesis of G.Shimura-Y.Taniyama:**
   All elliptic curves is modular curves
   Is wrong
2. **Proof of A.Wiles to Last Theorem of P.Fermat**
   Is doubtful.

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Introduction

It is well known that:

1. **David Hilbert, while solving the problem of Gordan's invariants, presented a universal formulation of this problem I following way:**

«Suppose, there is given an endless system of forms of a finite number of variables. Under whatcircumstances does a finite system of forms exist through which all others

areexpressed in the form of linear combinations are integral rational functions of the variables»

Universality of the given formulation lies in the fact thatit it containsin in a generalized form the drscription of a finalsolution of the Last Theorem Fermat's.

In our cause –this infinitely multitude equations:

$$a^n + b^n = c^n$$

each of which is realized at a concrete exponent of power n.

The number of the generalized variable is finite:

$$a,\ b,\ c,\ n\ .$$

In our case, use is made of three forms:

(E)

$$(a_0^{\ 2} \times a_0^{\ n-2}) + (b_0^{\ 2} \times a_0^{\ n-2}) = (c_0^{\ 2} \times a_0^{\ n-2})$$

$$(a_0^{\ 2} \times b_0^{\ n-2}) + (b_0^{\ 2} \times b_0^{\ n-2}) = (c_0^{\ 2} \times b_0^{\ n-2})$$

$$(a_0^{\ 2} \times c_0^{\ n-2}) + (b_0^{\ 2} \times c_0^{\ n-2}) = (c_0^{\ 2} \times c_0^{\ n-2})$$

based on the Pythagor equation:

$$a_0^{\ 2} + b_0^{\ 2} = c_0^{\ 2}$$

The integral rational functions of the variables appeared to be the proportionality coefficients:

$$S_a = a_0^{\ n-2}$$

$$S_b = b_0^{\ n-2}$$

$$S_c = c_0^{\ n-2}$$

Further on, let's add term by term the obtained equations (E) and arithmetically average these sums.

As a result, we will obtain one combined equatio

(F)

$$(a_0^{\ 2} \times D_n) = (b_0^{\ 2} \times D_{n)} + (c_0^{\ 2} \times D_n)$$

Here

$$D_n = (a_0^{\ n-2} + b_0^{\ n-2} + c_0^{\ n-2}) / 3$$

common multiplier end

$$(\ a_0\ ,\ b_0\ ,\ c_0\ )$$

primitive pythagora's triplets.

Usid equation (F), we mat write down the identification of its components:

$$a_*^{\,n} = a_0^{\,2} \times D_n$$

$$b_*^{\,n} = b_0^{\,2} \times D_n$$

$$c_*^{\,n} = c_0^{\,2} \times D_n$$

From these identification equations, we derive folloving formulas for determining roots:

$$a_* = \sqrt[n]{a_0^{\,2} \times D_n}$$

$$b_* = \sqrt[n]{b_0^{\,2} \times D_n}$$

$$c_* = \sqrt[n]{c_0^{\,2} \times D_n}$$

for the basis Fermat's equations :

$$a_*^{\,n} + b_*^{\,n} = c_*^{\,n}$$

and for the more general equations:

$$a^n + b^n = c^n$$

if

$$a = a_* \cdot k$$

$$b = b_* \cdot k$$

$$c = c_* \cdot k$$

at any integer multiplier k from an infinite series of natural numbers , see [8].

2.Secondary forms of Numbers Theory by H. Poincare include the definite algorithm of the proof of the Last Theorem by P. Fermat, see. [5] , [6] , [7].

3.In 1993, in Russia was published a book, [8] , in Russian and English languages. In this book the algorithm of geometrical proof of the Last theorem is described. Algorithm is based on 9 invariant triplets given in the book under numbers (1.6) – ((1.14). Those triplets are elements of secondary forms by H. Poincare. Completeness of my proof is characterized by the fact, that it (proof) is finished with formulas,see [3] , page7 , for calculation of all roots for Fermat's equation :

$$a_*^{\,n} + b_*^{\,n} = c_*^{\,n}$$

at all even and odd indicators of degree n .

4. **Hypothesis by Shimura-Taniyama is wrong and proof of A.Wilis is questionable because there is a great variety of non-modular elliptic curves information about which is in equations by G.Frey , [3]:**

$$Y^2 = (X - A) \times X \times (X + B) \quad (1)$$

This fact is easily illustrated wuth the help of equation of my elliptical curve:

$$Y^2 = a_0{}^n \times b_0{}^n \times c_0{}^n \quad (2)$$

which comes from equation of G.Frey at following substitutions:

$$\begin{aligned} (X - A) &= a_0{}^n \\ X &= b_0{}^n \\ (X + B) &= c_0{}^n \end{aligned} \quad (3)$$

Here:

$$\begin{aligned} a_0 &= v^2 - u^2 \\ b_0 &= 2vu \\ c_0 &= v^2 + u^2 \end{aligned} \quad (4)$$

primitive triads by Pifagora and $v > u$ are natural numbers of different eventy.

And:

$$n = 2 \text{ or } n > 2 \quad (5)$$

It is known, that Frey's curve demonstrates features which are deeply different from feature,see chapter X1.2 , paragraph A in the book [4].
I used this difference constructing my elliptical curve, see (2).
Unlike A.Willis, my method of proof is DEDUCTIVE.
I construct ready forms of decisions being led by INTUITION.
Virtue of this method is very well described in book by R.Courant and H.Robbins «What is Mathematics?», see beginning of the book [1].

Let's envisage properties of my curves.
Let's figure out MINIMAL DISCRIMINANT of the curve, see [4], for first primitive triad:

$$\begin{aligned} a_0 &= (2^2 - 1^2) = 3 \\ b_0 &= (2 \times 2 \times 1) = 4 \\ c_0 &= (2^2 + 1^2) = 5 \end{aligned} \quad (6)$$

at minimal n=2 :

$$\Delta = [(a_0 \times b_0 \times c_0)^{2n}] / 2^8 = 50625 \quad (7)$$

For simple n =5, minimal discriminant is equal to:

$$\Delta = 23.6196 \times 10^{14} \tag{8}$$

As far as discriminants are not equal to zero,
curves are NON-SINGULAR.
So those are ELLIPTICAL CURVES.

To this fact also refers the fact that simple n=2 DOES NOT DEVIDE its discriminant (7).

Experts know, why number 16 has a meaning of “litmus paper”
in theory of elliptical curves.
Without details let’s demonstrate this feature of number 16
on definite example for primitive triad (6).
At n=5 my curve gets determined expression:

$$Y^2 = 243 \times 1024 \times 3125 = 777600000 \tag{9}$$

At that:
16 devides 243 with oddment 3
16 devides 1024 with oddment 0
16 devides 3125 with oddment 5
16 divides number:

$$A = (b_0^{\ 5} - a_0^{\ 5}) = 1024^5 - 243^5 = 1125.0526 \times 10^{12}$$

with oddment, approximate, 5 .

It means that numbers forming
the given elliptical curve can’t
be compared by module d = 16 .

CONCLUSION :

MY ELLIPTICAL CURVES IS NON–MODULAR
HYPOTHESIS BY SHIMURA –TANIYAMA IS WRONG
PROOF OF A.WILIS IS UNCERTAIN

It is common problem of theory numbers :

GENERAL THEOREM

It is 9 TYPS of positive whole numbers :

$$(1) \quad \left| \begin{array}{l} N,\ (v > u),\ (a_0, b_0, c_0), \\ (3F'_a, 3F'_b, 3F'_c),\ (A, B, C), \\ (x, y, z),\ (p, q, r),\ n,\ (X^*, A^*, B^*) \end{array} \right.$$

as equivalent of 9 secondary reduction forms of H.Poincare ,

at which have to natural basis for solutions of the following system equations :

NATURAL BASIS

1,	2,	3,	4,	5,	6,	7,	8,	9,

u=1	v=2

2	9

$$[(1+1)=2]\times 2^{n}$$
$$[(1+2^{3})=3^{2}]\times(3^{3})^{n}$$

$$a_0 = v^2 - u^2$$
$$b_0 = 2vu$$
$$c_0 = v^2 + u^2$$

$$a_* = \sqrt[n]{a_o{}^2 \times D_n}$$
$$b_* = \sqrt[n]{b_o{}^2 \times D_n}$$
$$c_* = \sqrt[n]{c_o{}^2 \times D_n}$$
$$A^x + B^y = C^z$$
$$A^p x + B^q y = C^r z$$

$$a_*^{\,n} + b_*^{\,n} = c_*^{\,n}$$

$$Y^2 = (X^* - A^*)\times X^* \times (X^* + B^*) =$$
$$= a_0{}^2(3D_n)\times b_0{}^2(3D_n)\times c_0{}^2(3D_n) =$$
$$= 3F'_a \times 3F'_b \times 3F'_c$$

$$(a'_1\alpha'_1 + a'_2\,\alpha'_2 + a'_3\alpha'_3)/3F'_a = 1$$
$$(b'_1\beta'_1 + b'_2\beta'_2 + b'_3\beta'_3)/3F'_b = 1$$
$$(c'_1\gamma'_1 + c'_2\gamma'_2 + c'_3\gamma'_3)/3F'_c = 1$$

Here:
A, B, C , common multiplier for system equation

(2)
$$A^x + B^y = C^z$$
$$A^p x + B^q y = C^r z$$

and

numbers A^*

in equations for non-modular elliptic curves

(3)
$$Y^2 = (X^* - A^*) \times X^* \times (X^* + B^*)$$

WILL NOT DIVIDE INTO / BY

(4)
$$16 = 4 \times (b_0 = 2vu) = 4 \times (b_0 = 2 \times 2 \times 1)$$

if $v = 2$ and $u = 1$

over

first condition of secondary reduction forms H.Poincare:

(5)
$$a'_1\alpha'_1 + a'_2\alpha'_2 + a'_3\alpha'_3 = 1$$

in our case it is widened up to three corresponding conditions:

(6)
$$(a'_1\alpha'_1 + a'_2\alpha'_2 + a'_3\alpha'_3) / 3F'_a = 1$$
$$(b'_1\beta'_1 + b'_2\beta'_2 + b'_3\beta'_3) / 3F'_b = 1$$
$$(c'_1\gamma'_1 + c'_2\gamma'_2 + c'_3\gamma'_3) / 3F'_c = 1$$

I offer to you attention a solution of this problems as problem of General theorem arithmetic, [1] :

Every natural number N is either prime or can be uniquely factored as a product of primes in a unique way.

Primes = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...,1081,…. }

and as problem of Principle general (geometrical) co-variation, see [2].

P a r a d i g m

«Here is not the best place to come to detailed philosophical or psychological analysis of mathematics. Any where I'd like to stress a few moments. Excessive underlining of axiomatic- deductive character of mathematics seems to be very dangerous. Of course, beginning of any constructive creative work
(intuitive origin) is a source of our ideas and arguments, hardly keeps within simple philosophical formula; and anywhere just this origin is a genuine core of any mathematical discovery, even if it belongs to the most abstract spheres. If a target is a clear deductive form, so the motive of mathematics is intuition and construction.», [1]. Being directed by this initial position of outstanding mathematician R. Curant I offer to readers attention a result of intuitive construction from which can follow any axiomatic-deductive constructions. Intuitive -constructive origin is always very simple. Here is the beginning of such constructions:

PART № 1
Non-modular elliptic curves

It is natural numbers:
N = 1, 2, 3, 4, 5, 6, 7, …., ∞
the quantity of the variables numbers N is endless series.

If numbers v > u are the numbers of various evenness taken from endless series of natural numbers, then we hawe:

1. **The natural numbers N,**

2. **The primitive Pythagorean triplets**

(7)
$$\begin{aligned} a_0 &= v^2 - u^2 \\ b_0 &= 2vu \\ c_0 &= v^2 + u^2 \end{aligned}$$

3. **The primitive Diophantine triplets**

(8)
$$\begin{aligned} a_* &= \sqrt[n]{a_0{}^2 \times D_n} \\ b_* &= \sqrt[n]{b_0{}^2 \times D_n} \\ c_* &= \sqrt[n]{c_0{}^2 \times D_n} \end{aligned}$$

is are functionally independent.

Here:

(9)
$$D_n = (a_0{}^{n-2} + b_0{}^{n-2} + c_0{}^{n-2}) / 3$$

common multiplier as generalized proportionality coefficient

Comment

Triplets (a_*, b_*, c_*) is roots for basis equation of Fermat:

(10)
$$a_*{}^n + b_*{}^n = c_*{}^n$$

THEOREM № 1

According of Principle general (geometrical) co-variation , [2], elliptic curve

(11)
$$Y^2 = X(X - A)(X + B)$$

be of universal geometrical equivalent, see Fig.1:

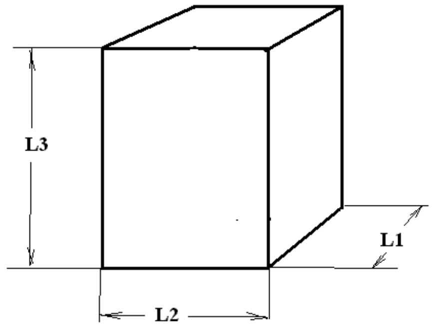


Fig.1

Here:

(12) $$L1=(X-A)\ ;\quad L2=X\ ;\quad L3=(X+B)\ ;$$

If

$$\left|\begin{array}{l} L1=a^n \\ L2=b^n \\ L3=c^n \end{array}\right.$$

then we have the following mathematical construction for the capacity:

(13) $$a^n\times b^n\times c^n$$

and the following mathematical construction:

(14) $$a^2\times b^2\times c^2$$

as construction for equation of Pythagoras:

(15) $$a^2+b^2=c^2$$

It is concrete realization of Principle general (geometrical) co-variation.

THEOREM № 2

Equation of Diophant - Fermat:

(16) $$a^n+b^n=c^n$$

is equivalent for equation Pythagoras, if n = 2 :

(17) $$a^2+b^2=c^2$$

and

(18) $$\begin{aligned} Y^2 &= (X^*-A^*)\times X^*\times(X^*+B^*)= \\ &= {a_0}^n\times {b_0}^n\times {c_0}^n \end{aligned}\quad ,$$

equation non-modular elliptic curve:

if (a,b,c) :

(19)
$$\left|\begin{array}{l} a = a_0 = (v^2 - u^2) \\ b = b_0 = (2vu) \\ c = c_0 = (v^2 + u^2) \end{array}\right.$$

0R

(20)
$$\left|\begin{array}{l} a = S \times a_0 = S \times (v^2 - u^2) \\ b = S \times b_0 = S \times (2vu) \\ c = S \times c_0 = S \times (v^2 + u^2) \end{array}\right.$$

is Pythagorean triplets
and

(21)
$$v > u$$

is numbers of various evenness taken from endless series of natural numbers.

If , see (18):

$$(X^* - A^*) = a_0^{\ n}$$
$$X^* = b_o^{\ n}$$
$$(X^* + B^*) = c_0^{\ n}$$

then we have non-modular curve

Here numbers

$$A^* = (b_0^{\ n} - a_0^{\ n})$$

WILL NOT DIVIDE INTO

(22)
$$16 = 4 \times (b_0 = 2vu) = 4 \times (b_0 = 2 \times 2 \times 1)$$

Only one thing can be deduced:

Hypothesis of Shymura-Thaniyama
«All elliptic curves are modular curve»,[3],
is notauthentic hypothesis .
According
proof of A.Wiles, [4] , is unauthentic proof .

C O N S E Q U E N C E № 1

Here all numbers is reciprocals primes:

(23)
$$\begin{cases} Y^2 = a_0{}^2 \times b_0{}^2 \times c_0{}^2 = \\ = (X^* - A^*) \times X^* \times (X^* + B^*) \\ \\ Y = a_0 \times b_0 \times c_0 = \\ = \Psi[(v^2 - u^2)(2vu)(v^2 + u^2)] \end{cases}$$

C O N C E Q U E N C E № 2

According Fundamental theorem arithmetic this equality

(24)
$$(a_* \times b_* \times c_*)^n = \\ = (a_0 \times b_0 \times c_0)^n = \\ = Y^2$$

contents all non-integer roots

$$(a_*{}^n, b_*{}^n, c_*{}^n)$$

for equations:

(25)
$$a_*{}^n + b_*{}^n = c_*{}^n$$

(26)
$$a^n + b^n = c^n$$

Here:

(27)
$$\begin{cases} a = a_* \times S \\ b = b_* \times S \\ c = c_* \times S \end{cases}$$

and S common multiplier.

E X P L A N A T O R Y E X A M P L E

We hawe primitive Pythagora's triplet :

(28)
$$\begin{cases} a_0 = 12 \\ b_0 = 35 \\ c_0 = 37 \end{cases}$$

according (18) :

(29)
$$\begin{aligned} Y^2 &= a_0{}^2 \times b_0{}^2 \times c_0{}^2 = \\ &= 144 \times 1225 \times 1369 = \\ &= 241491600 \end{aligned}$$

and

(30)
$$Y = a_0 \times b_0 \times c_0 = 15540$$

(31)
$$\begin{aligned} &(X^* - A^*) = a_0{}^2 = 144 \\ &X^* = b_0{}^2 = 1225 \\ &(X^* + B^*) = c_0{}^2 = 1369 \end{aligned}$$

(32)
$$\begin{aligned} B^* &= 1369 - X^* = \\ &= 1369 - 1225 = 144 \end{aligned}$$

(35)
$$\begin{aligned} A^* &= X^* - 144 = \\ &= 1225 - 144 = 1081 \end{aligned}$$

Here number

$A^* = 1081$

WILL NOT DIVIDE INTO

(36)
$$16 = 4 \times (b_0 = 2vu) = 4 \times (b_0 = 2 \times 2 \times 1)$$

PART № 2

Two type of non-modular elliptic curves:

(A) Non-modular elliptic curves of the first type

(37)
$$\begin{aligned} Y^2 &= (X^* - A^*) \times X^* \times (X^* + B^*) = \\ &= a_0{}^2 \times b_0{}^2 \times c_0^2 \end{aligned}$$

(38)
$$Y = a_0 \times b_0 \times c_o$$

(39)
$$\left|\begin{array}{l}(X^* - A^*) = a_0{}^2 \\ X^* = b_0{}^2 \\ (X^* + B^*) = c_0{}^2\end{array}\right.$$

(B) Non-modular elliptic curves of the second type

(40)
$$\begin{aligned}Y^2 &= (X^* - A^*) \times X^* \times (X^* + B^*) = \\ &= a_0{}^2(3D_n) \times b_0{}^2(3D_n) \times c_0{}^2(3D_n) = \\ &= 3F'_a \times 3F'_b \times 3F'_c\end{aligned}$$

(41)
$$\left|\begin{array}{l}(X^* - A^*) = a_0{}^2(3D_n) = 3F'_a \\ X^* = b_0{}^2(3D_n) = 3F'_b \\ (X^* + B^*) = c_0{}^2(3D_n) = 3F'_c\end{array}\right.$$

Here number A^*

WILL NOT DIVIDE INTO

(42)
$$16 = 4 \times (b_0 = 2vu) = 4 \times (b_0 = 2 \times 2 \times 1)$$

if $v = 2$ and $u = 1$.

and according (38) :

(43)
$$\left|\begin{array}{l}a_* = \sqrt[n]{F'_a} = \sqrt[n]{a_0{}^2 \times D_n} \\ b_* = \sqrt[n]{F'_b} = \sqrt[n]{b_0{}^2 \times D_n} \\ c_* = \sqrt[n]{F'_c} = \sqrt[n]{c_0{}^2 \times D_n}\end{array}\right.$$

is roots for basis equation by Fermat

(44)
$$a_*{}^n + b_*{}^n = c_*{}^n$$

and

(45)
$$\left|\begin{array}{l}F'_a = a'_1 \times \alpha'_1 = a'_2 \times \alpha'_2 = a'_3 \times \alpha'_3 \\ F'_b = b'_1 \times \beta'_1 = b'_2 \times \beta'_2 = b'_3 \times \beta'_3 \\ F'_c = c'_1 \times \gamma'_1 = c'_2 \times \gamma'_2 = c'_3 \times \gamma'_3\end{array}\right.$$

is elements of secondary reduction forms
of the theory numbers by H.Poincare [5], [6], [7], [8], [9], [10], [11].

PART № 3

SECONDARY REDUCTION FORMS OF H.POINCARE'S THEORY OF NUMBERSEXACTLY SOLVE FERMAT'S EQUATION AT ALL $n \geq 2$

In the first publication [5] of theory of numbers, in the introduction, Poincare marks the following:
«Arithmetic research of homogenous forms is one of the most interesting questions of theory of numbers and of the questions , which are most interesting for geometrysts.» See. [5].
This statement of Poincare is enough realized, but not formulated yet, Principle of universal co-variation . See. [2] .

The aim of this article is to confirm geometric essence of one special paragraph of Poincare's theory of numbers which contains information about exact geometric proof of The Last theorem by Fermat.

For that purpose let's apply to publication [6] by Poincare, translated into Russia , see. [7]
Here I give a quotation [7] :

«Everything we have spoken before, can be applied only to main reduction forms, so to them we can give the following results:
1) **Each class commonly saying is only one main reduction form;**
2) **There are infinitely many classes;**
3) **Main reduction forms are divided into three types;**
4) **Form of the first and second type is a finite number;**
5) **Forms of the third type are divided into infinite multitude of sorts, and each sort contains infinitely mane reduction forms.**

Let's attend to secondary reduction forms.»

From analysis of these forms let's choose a fragment which has a link to geometric proof of The Last theorem by Fermat see. [8] and [9] .
This fragment Poincare explains the following way, see p.889 in [6] :

«As three simple numbers a_1, a_2, a_3 are mutually simple, there are always Nine Whole Numbers, satisfying the following conditions:

$$
(46) \qquad \left|
\begin{aligned}
&a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 1 \\
&a_1 = \beta_2\gamma_3 - \beta_3\gamma_2 \\
&a_2 = \beta_3\gamma_1 - \beta_1\gamma_3 \\
&a_3 = \beta_1\gamma_2 - \beta_2\gamma_1
\end{aligned}
\right.
$$

Then instead of substitutions, used by Poincare we will use substitutions by Frei , see. [3] , which uses Frei at research of features of his elliptical curve:

$$Y^2 = X(X - A)(X + B) \tag{47}$$

As in Poincare's, here are used mutually simple numbers.
Then we get convinced that substitutions by Frei in equation (47) correspond to one of the reduction forms, number of which in each sort of forms of the Third type is infinite. We will get convinced that my substitutions see (50), widening substitutions of Frei, brings researcher to geometrical proof of the Last theorem by Fermat (LTF), which reader can find in [8] and [9].

Frei uses the following substitutions:

$$A = a^q \quad \text{and} \quad B = b^q \tag{48}$$

As far as Fermat's equation:

$$a^n + b^n = c^n \tag{49}$$

contains three members, the author [8] forms not two, but three substitutions, changing simple number q for any whole number $n \geq 2$:

$$\left|\begin{aligned} A &= a^n \\ B &= b^n \\ C &= c^n \end{aligned}\right. \tag{50}$$

In the result Fernat's equation gets simple phenomenologic formulas for calculation of its PRIMITIVE SOLUTIONS at any index of degree $n \geq 2$:

$$\left|\begin{aligned} a_* &= \sqrt[n]{A} \\ b_* &= \sqrt[n]{B} \\ c_* &= \sqrt[n]{C} \end{aligned}\right. \tag{51}$$

Any non-primitive solutions of Fermat's equation are calculated by simple multiplying of primitive solutions to any common multiplier S :

$$\left|\begin{aligned} a &= a_* \times S \\ b &= b_* \times S \\ c &= c_* \times S \end{aligned}\right. \tag{52}$$

Further widening of infinite number of calculated solutions of Fermat's equation are done with the help of any root multiplier, including special multiplier:

(53) $$D_n = (a_0^{\,n-2} + b_0^{\,n-2} + c_0^{\,n-2})/3$$

In this case we get universal forms for solution of Fermat's equation:

(54) $$\left|\begin{array}{l} a = \sqrt[n]{A \times D_n} \\ b = \sqrt[n]{B \times D_n} \\ c = \sqrt[n]{C \times D_n} \end{array}\right.$$

Construction of calculated solutions of equation by Fermat
Is finished with intuitive construction of rtiad of mutually
simple numbers A, B, C :

(55) $$\left|\begin{array}{l} A = a_0^{\,2} \\ B = b_0^{\,2} \\ C = c_0^{\,2} \end{array}\right.$$

Here as in construction of multiplier D_n,
are used primitive Pythagora's triads:

(56) $$\left|\begin{array}{l} a_0 = v^2 - u^2 \\ b_0 = 2vu \\ c_0 = v^2 + u^2 \end{array}\right.$$

being constructed from any pair $v > u$ of natural numbers
of different evenness.

To tie up take by us calculated solutions of the equation by Fermat with substitutions of Poincare , let's remember mentioned above remark
of Poincare that arithmetical theory of numbers
has geometrical interpretation .

Described above formula for calculation of roots for Fermat's equation also have geometrical interpretation .
This interpretation is described in [8] and [9].

The interpretation is based on building of nine triads of rectangles - squares of Diophant. Every three triads of squares make indivisible geometrical variety consisting of three isometric triangles of Diophant.

Below I give an illustration of the above said algorithm
with the help of Fig.2 , taken from [8] and [9] :

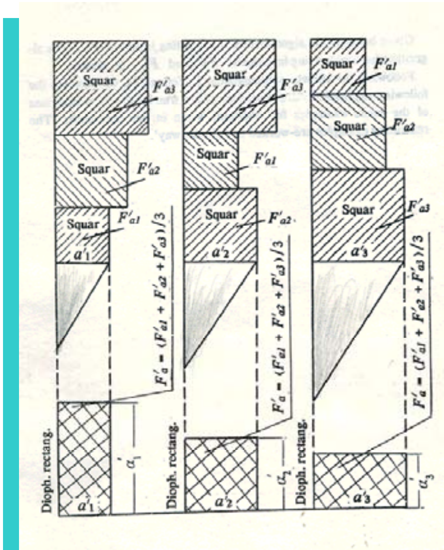


Fig. 10. Three variants of construction of equidimensional in area Diophantine rectangles, yielding the same result — smaller invariant F'_a of Diophantine space

Fig.2

On this picture is given a construction of three triads of triangles - squares of Diophant, which comes to construction of three isometric areas F'_a of triangles of Diophant. The area F'_a is the smallest invariant of Diophant, defining the smallest root for Fermat's equation.

The same way are made medium and biggest invariants which define medium and biggest roots for Fermat's equation:

$$(57) \qquad \left| \begin{array}{l} F'_a = a_0{}^2 \times D_n \\ F'_b = b_0{}^2 \times D_n \\ F'_c = c_0{}^2 \times D_n \end{array} \right.$$

Below, see Fig.2, we see three isometric by area F'_a triangles of Diophant. Above, on the cathetus of corresponding rectangular triangles are made three triads of own rectangles-squares of Diophant, medium arithmetic meaning of the areas of which is equak to three isometric areas of the corresponding rectangles of Diophant, drawn in the bottom of Fig. 2 .

The result of such geometric constructions is a construction of three MAIN algebraic invariants , see.(57). From Fig.2 comes construction of the FIRST (smallest) invariant in which are used marks of sides of the rectangle drawn in the bottom of the drawing :

FIRST (smallest) invariant

$$(58) \qquad F'_a = a'_1 \times \alpha'_1 = a'_2 \times \alpha'_2 = a'_3 \times \alpha'_3$$

The same way are made
SECOND (medium) invariant:

(59) $$F'_b = b'_1 \times \beta'_1 = b'_2 \times \beta'_2 = b'_3 \times \beta'_3$$

and THIRD (biggest) invariant :

(60) $$F'_c = c'_1 \times \gamma'_1 = c'_2 \times \gamma'_2 = c'_3 \times \gamma'_3$$

In these geometric models the base of the rectangles of Diophant is equal to smallest cathetus of corresponding rectangular triangles areas of which are not isometric see Fig.2 .
Invariants (58) – (60) are connecting links between forms of Poincare see. (46) , and forms (50) , made according to forms of Frei (48) .
At last these invariants make the basis for formulas (54), with the help of which are calculated roots of Fermat's equation (49) .

Proof of the above said .
According to [8] and [9] in space of Diophant's variety one can build NINE invariant algebraic forms, numeric meanings of which are defined HIGHTS of Diophant's rectangles , see Fig.2:

(61)
$$\left|\begin{array}{l} \alpha'_1 = (a_0 \times D_n) / \sqrt{a_0^{\,n-2}} \\ \alpha'_2 = (a_0 \times D_n) / \sqrt{b_o^{\,n-2}} \\ \alpha'_3 = (a_0 \times D_n) / \sqrt{c_0^{\,n-2}} \end{array}\right.$$

(62)
$$\left|\begin{array}{l} \beta'_1 = (b_0 \times D_n) / \sqrt{a_0^{\,n-2}} \\ \beta'_2 = (b_0 \times D_n) / \sqrt{b_0^{\,n-2}} \\ \beta'_3 = (b_o \times D_n) / \sqrt{c_0^{\,n-2}} \end{array}\right.$$

(63)
$$\left|\begin{array}{l} \gamma'_1 = (c_0 \times D_n) / \sqrt{a_o^{\,n-2}} \\ \gamma'_2 = (c_0 \times D_n) / \sqrt{b_0^{\,n-2}} \\ \gamma'_3 = (c_0 \times D_n) / \sqrt{c_0^{\,n-2}} \end{array}\right.$$

At that, bases of Diophant's rectangles, see Fig.10, are SEGMENTS LENGTHS of which correspondingly are:

(64)
$$\begin{cases} a'_1 = a_0 \times \sqrt{a_0^{\,n-2}} \\ a'_2 = a_0 \times \sqrt{b_0^{\,n-2}} \\ a'_3 = a_0 \times \sqrt{c_0^{\,n-2}} \end{cases}$$

(65)
$$\begin{cases} b'_1 = b_0 \times \sqrt{a_0^{\,n-2}} \\ b'_2 = b_0 \times \sqrt{b_0^{\,n-2}} \\ b'_3 = b_0 \times \sqrt{c_0^{\,n-2}} \end{cases}$$

(66)
$$\begin{cases} c'_1 = c_0 \times \sqrt{a_0^{\,n-2}} \\ c'_2 = c_0 \times \sqrt{b_0^{\,n-2}} \\ c'_3 = c_0 \times \sqrt{c_o^{\,n-2}} \end{cases}$$

Let's pay attention that MAIN invariants, see. (58) – (60), are constructed from invariants (61) – (66) .

At last we have come to a final.
All the chain of described above substitutes is closed on conditions conditions of Poincare's substitutes ,see. (46) .

First condition:

(67)
$$a'_1\alpha'_1 + a'_2\alpha'_2 + a'_3\alpha'_3 = 1$$

in our case it is widened up to three corresponding conditions:

(68)
$$\begin{cases} (a'_1\alpha'_1 + a'_2\,\alpha'_2 + a'_3\alpha'_3)/3F'_a = 1 \\ (b'_1\beta'_1 + b'_2\beta'_2 + b'_3\beta'_3)/3F'_b = 1 \\ (c'_1\gamma'_1 + c'_2\gamma'_2 + c'_3\gamma'_3)/3F'_c = 1 \end{cases}$$

At that, formation of Poincare's conditions, given in the beginning of the article :

«As three whole numbers a_1, a_2, a_3 are mutually simple , there are always NINE whole numbers , satisfying the following conditions :

(69)
$$\left|\begin{array}{l} a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 1 \\ a_1 = \beta_2\gamma_3 - \beta_3\gamma_2 \\ a_2 = \beta_3\gamma_1 - \beta_1\gamma_3 \\ a_3 = \beta_1\gamma_2 - \beta_2\gamma_1 \end{array}\right.$$

make sense , coordinated with geometrical proof of the Last theorem by Fermat:

So if three whole numbers

(70)
$$a_0 , b_o , c_o$$

making each triad of primitive Pythagora's numbers, are mutually simple, it means that at all indexes of degree n ,
there are always NINE WHOLE NUMBERS:

(71)
$$\left|\begin{array}{l} a_1 = a_1'^2 = a_0^{\,2}(a_0^{\,n-2}) \\ a_2 = a_2'^2 = a_0^{\,2}(b_0^{\,n-2}) \\ a_3 = a_3'^2 = a_0^{\,2}(c_0^{\,n-2}) \end{array}\right.$$

(72)
$$\left|\begin{array}{l} b_1 = b_1'^2 = b_0^{\,2}(a_0^{\,n-2}) \\ b_2 = b_2'^2 = b_0^{\,2}(b_0^{\,n-2}) \\ b_3 = b_3'^2 = b_0^{\,2}(c_0^{\,n-2}) \end{array}\right.$$

(73)
$$\left|\begin{array}{l} c_1 = c_1'^2 = c_0^{\,2}(a_0^{\,n-2}) \\ c_2 = c_2'^2 = c_0^{\,2}(b_0^{\,n-2}) \\ c_3 = c_3'^2 = c_0^{\,2}(c_0^{\,n-2}) \end{array}\right.$$

that satisfy three conditions (68).

At that three other conditions of Poincare turn into zero :

(74)
$$\left|\begin{array}{l} \beta_2'\gamma_3' - \beta_3'\gamma_2' = 0 \\ \beta_3'\gamma_1' - \beta_1'\gamma_3' = 0 \\ \beta_1'\gamma_2' - \beta_2'\gamma_1' = 0 \end{array}\right.$$

As a result Fermat's equation, see (46), gets easily calculated formula, calculation of which comes to calculation of areas of rectangles of Diophant, that means calculation of MAIN invariants (58) – (60) .

In this case roots of Fermat's equation (46) are calculated with the help of the following formulas:

$$\text{(75)} \qquad \left| \begin{aligned} a &= \sqrt[n]{F'_a} \\ b &= \sqrt[n]{F'_b} \\ c &= \sqrt[n]{F'_c} \end{aligned} \right.$$

$$\text{(76)} \qquad \left| \begin{aligned} a &=: a \times S \\ b &=: b \times S \\ c &=: c \times S \end{aligned} \right.$$

These formulas as it was shown above, have direct and reverse connection with theory of numbers of H.Poincare and geometrical variety of Diophant and Pythagora.

All said above corresponds to Principle of universal (geometrical) co-variation, which is a basis of all fundamental research of the twentieth century, and simple geometric proofs of the Last theorem of Fermat, which don't need any use of elliptic curves and modular forms .

If reader gets acquainted with web-sites
http://yvsevolod-26.narod.ru/index.html
http://yvsevolod-29.narod.ru/index.html
http://yvsevolod-28.narod.ru/index.html
kept in the Narod catalogues of Russian Internet,
he will get convinced in simple but easily proved truth:

Harmony of space, harmony of life on the Earth and Universe are reflected in great harmony of natural numbers so capaciously and variously described in Theory of Numbers by H.Poincare.

Note: Correctness of given here substitutes connecting one of the secondary given forms of Poincare with geometric proof of LTF ,can be checked by easy calculations on pocket calculator making any pair (v > u)of natural numbers of different evenness and calculating with a formula (56) corresponding primitive triad of Pythagora..

PART № 4

ALGORITHM-PROOF CONJECTURE BEAL

It is problem,
formed by A. Beal :

Let's take A, B, C, x, y, z are positive whole numbers at which
x, y, z > 2 .

If there are solutions of this equation

(77) $$\mathbf{A^x + B^y = C^z}$$

then

A, B, C have a common multiplier .

I offer to you attention a solution of this problem as solution of system of equations by A. Beal and P.Fermat :

(78) $$\left|\begin{array}{l} \mathbf{a^n + b^n = c^n} \\ \mathbf{A^x + B^y = C^z} \\ \mathbf{A^p x + B^q y = C^r z} \end{array}\right.$$

Algorithm-Solution of this system of equations supports mathematical foreknowledge by A.Beal and connection of this foreknowledge with ELEMENTARY ALGORITHM-PROOF of the last theorem by P.Fermat.

Theorem

System of equations:

(79) $$\left|\begin{array}{l} \mathbf{a^n + b^n = c^n} \\ \mathbf{A^x + B^y = C^z} \\ \mathbf{A^p x + B^q y = C^r z} \end{array}\right.$$

has a base solution in a form of identical equality of two natural numbers :

(80) $$\left|\begin{array}{l} \mathbf{2 = 2} \\ \mathbf{9 = 9} \end{array}\right.$$

infinite enhancement of base solution (80) is executed at the expense of infinite variety of common multipliers:

(81) $$\left|\begin{array}{l} \mathbf{2 \times S_2} \\ \mathbf{9 \times S_3} \end{array}\right.$$

that have invariant view:

(82)
$$\begin{vmatrix} S_2 = 2^n \\ S_3 = (3^3)^n \end{vmatrix}$$

at any whole meaning of n in the second case and at any even meaning of n in the first case.

PROOF OF THE THEOREM

Base solution (80) of the system (79) has a number of equivalent views:

(83)
$$\begin{vmatrix} [2 = (1+1)] = (3-1) \\ 9 = (3+3+3) = [(1+2^3) = 3^2] \end{vmatrix}$$

That transforms enhancement (81) of the base solution (80) into two fundamental forms of theory of numbers :

(84)
$$\begin{vmatrix} [(1+1) = 2] \times S_2 \\ [(1+2^3) = 3^2] \times S_3 \end{vmatrix}$$

These forms, taking into consideration (82) , come to easily calculated view:

(85)
$$\begin{vmatrix} [(1+1) = 2] \times 2^n \\ [(1+2^3) = 3^2] \times (3^3)^n \end{vmatrix}$$

Each infinite variety of calculated solutions (85) can be infinitely enhanced at the expense of multiplying for any natural number N form infinite variety of natural numbers:

(86)
$$\begin{vmatrix} \{ [(1+1) = 2] \times 2^n \} \times N \\ \{ [(1+2^3) = 3^2] \times (3^3)^n \} \times N \end{vmatrix}$$

It is easy to mention the role of common multiplier N in formulas (86) can take any rational or any irrational number.
According to Gedel theorem of incompleteness, any mathematical proof should be comcluded with formulas for calculation of the proven .
Formulas (85) and (86) meet objectives of Gedel theorem.

Let's demonstrate effectiveness of formulas (85) and (86).
First let's see their application for solution of A. Beal equation on definite examples.

Example 1.

We have equation

$$A^x + B^y = C^z$$

Considering n = 1 , then , see (85), we find:

$$S_3 = (3^3)^1$$

$$[(1+2^3) = 3^2]\times 3^3$$

That is equivalent to equation:

$$3^3 + 6^3 = 3^5,$$

Here A=3, B=6, C=3, x=3, y=3, z=5
Common multipliers in which are numbers

$$3 \quad \text{and} \quad 3^3$$

According to (86) we calculate :

$$(3^3 + 6^3 = 3^5)\times N$$

Example 2

We have equation

$$A^x + B^y = C^z$$

Considering n = 2, then ,see (85) ,we find:

$$S_3 = (3^3)^2$$

$$[(1+2^3) = (3^2)]\times 3^6$$

That is equivalent to equation:

$$3^6 + 18^3 = 3^8$$

Here A=3 , B=18 , C=3, x=6, y=3, z=8 .
Common multipliers in which are numbers:

$$3 \quad \text{and} \quad 3^6$$

According to (86) we calculate:

$$(3^6 + 18^3 = 3^8)\times N$$

Example 3

We have equation

$$A^x + B^y = C^z$$

Considering n = 3, then . see (85) ,we find:

$$S_3 = (3^3)^3$$

$$[(1+2^3) = 3^2] \times 3^9$$

That is equivalent to equation:

$$3^9 + 54^3 = 3^{11}$$

Here A=3, B=54, C=3, x=9, y=3, z=11 .

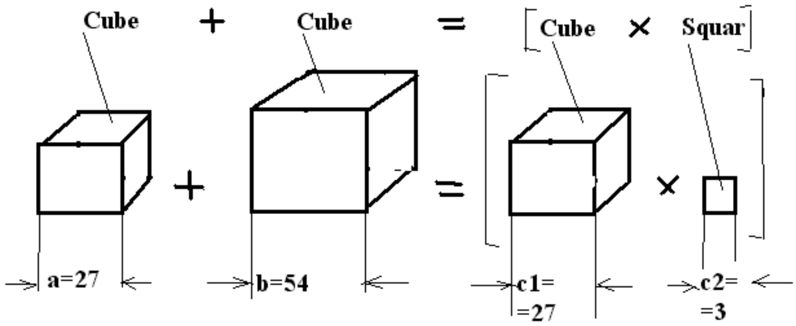
Common multipliers in which are numbers:

3 **and** 3^9

According to (86) we calculate:

$$(3^9 + 54^3 = 3^{11}) \times N$$

According we have geometrical and arithmetical models of example 3 :



$$27^3 + 54^3 = [27^3 \times 3^2]$$

Example 4

We have equation

$$A^x + B^y = C^z$$

Considering n = 4 , then ,see (85) ,we find :

$$S_3 = (3^3)^4$$

$$[(1+2^3) = 3^2] \times 3^{12}$$

That is equivalent to equation :

$$3^{12} + 162^3 = 3^{14}$$

Here A=3, B=162, C=3, x=12, y=3, z=14 .

Common multipliers in which are numbers :

3 **and** 3^{12}

According to (86) we calculate :

$$(3^{12} + 162^3 = 3^{14}) \times N$$

Example 5

We have equation

$$A^x + B^y = C^z$$

Considering n = 8, then , see (85) ,we find :

$$S_2 = 2^8$$

$$[(1+1) = 2] \times 2^8$$

That is equivalent to equation :

$$2^8 + 4^4 = 2^9$$

Here A=2, B=4, C=2, x=8, y=4, z=9 .

Common multipliers in which are numbers :

2 **and** 2^8

According to (86) we calculate :

$$(2^8 + 4^4 = 2^9) \times N$$

Example 6

We have equation

$$A^x + B^y = C^z$$

Considering n = 50 , then , see (85) ,we find :

$$S_3 = (3^3)^{50} = 3^{150}$$

$$[(1+2^3) = 3^2] \times 3^{150}$$

That is equivalent equation :

$$9^{75} + (1{,}435796 \times 10^{24})^3 = 3^{152}$$

Here $A = 9$, $B = (1{,}435796 \times 10^{24})$, $C = 3$, $x = 75, y = 3, z = 152$

Common multipliers in which are numbers :

3 **and** 3^{150}

According to (86) we calculate

$$[9^{75} + (1{,}435796 \times 10^{24})^3 = 3^{152}] \times N$$

Example 7

We have equation

$$A^x + B^y = C^z$$

Considering n=61, then, see (85) , we find :

$$S_3 = (3^3)^{61} = 3^{183}$$

$$[(1+2^3) = 3^2] \times 3^{183}$$

That is equivalent equation :

$$3^{183} + (2{,}5434695 \times 10^{29})^3 = (243)^{37}$$

Here A=3 , B= $2{,}5434695 \times 10^{29}$ **, C=243, x=183, y=3, z=37.**

Common multipliers in which are numbers:

3 and 3^{183}

Exaple 8 ,
[see (92) – (99)] .

We have basis equation of P.Fermat

(87) $$a_*^{\,n} + b_*^{\,n} = c_*^{\,n}$$

If

$$n = 8\,,\ v = 13\,,\ u = 6\ ,\ S_3 = (3^3)^8 = 3^{24}\,,$$

see (82) and (85) ,

then we calculate natural numbers of Pythagoras :

(88) $$\left|\begin{array}{l} a_0 = v^2 - u^2 = 133 \\ b_0 = 2vu = 156 \\ c_0 = v^2 + u^2 = 205 \end{array}\right.$$

and irrational roots for basis equation (87) of P.Fermat:

(89) $$\left|\begin{array}{l} a_* = \sqrt[n]{a_0^{\,2} \times S_3} = \sqrt[8]{17689 \times 3^{24}} = \\ = 3895{,}857535 \\ \\ b_* = \sqrt[n]{b_0^{\,2} \times S_3} = \sqrt[8]{24336 \times 3^{24}} = \\ = 4054{,}350645 \\ \\ c_* = \sqrt[n]{c_0^{\,2} \times S_3} = \sqrt[8]{42025 \times 3^{24}} = \\ = 4340{,}888414 \end{array}\right.$$

Comment - see (92) –(99) .

ET CETERA, ET CETERA …….

All given above examples support the conclusion, coming out of fundamental forms (85) and (86) of numbers theory:

NUMBER OF SOLUTIONS , WHICH MEET CONDITIONS OF THE THEOREM – HYPOTHESIS BY A. BEAL , URGES TOWARDS INFINITY.

See at that:
http://yvsevolod-28.narod.ru/index.html

What contradicts this conclusion is a conclusion of authors of the following publication:
H.Darmon and A.Granville, On the equations $z^m = F(x,y)$ and $Ax^p + By^q = Cz^r$. Bull. London. Math. Soc.27(1995), 513-543. See [12] .

Authors of this publication think that there is a limited variety of solutions , INDIRECTLY supporting fairness of A.Beal hypothesis .

Authors give ten examples, quasi supporting their conclusion. Among these examples there is the following one:

$$\left| \begin{array}{l} 1+2^3=3^2 \\ 2^5+7^2=3^4 \\ 7^3+13^2=2^9 \\ 2^7+17^3=71^2 \\ 3^5+11^4=122^2 \\ 17^7+76271^3=21063928^2 \\ 1414^3+2213459^2=65^7 \\ 43^8+96222^3=30042907^2 \\ 9262^3+15312283^2=113^7 \\ 33^8+1549034^2=15613^3 \end{array} \right. \tag{90}$$

But in this examples there are no common multipliers A, B and C . So they have nothing in common with common problem of A. Beal and P. Fermat.

Among 10 examples we find :

$$1+2^3=3^2 \tag{91}$$

They couldn't see in this example a simple decomposition of number 9 into original natural numbers (1,2,3), as I have done. See at that (80), (81), (82) and (50). Just from this decomposition starts elementary algorithm of numbers theory, that led me to final solution of system of equations A.Beal – P.Fermat, see. (79).

If we apply to my formulas (85) , we will find an example as a part of my universal formulas.
So, the mentioned authors were very close to common positive decision of the A.Beal problem, but their original (rather complicated) mathematical model failed in the very beginning .

PRIMARY CONDITIONS OF THE TASK WERE NOT ADEQUATE TO PRIMARY CONDITIONS OF HYPOTHESIS BY A.BEAL

I explain it with the fact that, pure mathematics, including theory of numbers, doesn't use opportunities of Principle of general (geometrical) co-variability by Pifagorus at which numbers (1, 2, 3) play main role in all disciplines of natural sciences .

See also:
http://yvsevolod-26.narod.ru/index.html
http://yvsevolod-27.narod.ru/index.html

Let's have a look at system of equations (79) from the point of view of P.Fermat problem.

THE SYSTEM EQUATIONS OF P. FERMAT HAS TWO LEVELS OF SOLUTION :

-Primary, consisting of infinite variety of whole-numbered Solutions,

,

-Secondary, consisting of infinite variety of Non-Whole-numbered solutions.

Non-Whole-numbered solutions of the system are functions of solutions of whole-numbered .

These solutions make infinite variety of irrational numbers , that make decisions of the famous equation:

(92) $$a^n + b^n = c^n$$

The basis of the system as it was shown above, are three first numbers of the natural numbers row, see (50). These three numbers make an amount :

(93) $$1 + 2 = 3$$

On which is base the principle of general (geometrical) co-variability by Pythagoras.
See also:
http://yvsevolod-27.narod.ru/index.html

Reference: This is a copy of the first page of my personal site .

Triads (a,b,c) in system (79) and in equation (92) are functions of natural row numbers , calculated with the help of simple formulas:

(94)
$$\begin{cases} a = a_* \times S \\ b = b_* \times S \\ c = c_* \times S \end{cases}$$

In which S is any whole-numbered common multiplier, and triads of numbers (a_*, b_*, c_*) are solutions of infinite variety of basis equations , see Example 8 :

(95)
$$a_*^{\ n} + b_*^{\ n} = c_*^{\ n}$$

Solutions of these equations are calculated with the help of my formulas:

(96)
$$\begin{cases} a_* = \sqrt[n]{a_0^{\ 2} \times D_n} \\ b_* = \sqrt[n]{b_0^{\ 2} \times D_n} \\ c_* = \sqrt[n]{c_0^{\ 2} \times D_n} \end{cases}$$

At any whole-numbered $n \geq 2$

Here:

(97)
$$D_n = (a_0^{\ n-2} + b_0^{\ n-2} + c_0^{\ n-2})/3$$

COMMON MULTIPLIER

Triads of numbers (a_0, b_0, c_0) are primitive Pythagoras triiads.
Primitive Pythagoras triads are calculated with the help of famous formulas:

(98)
$$\begin{cases} a_0 = v^2 - u^2 \\ b_0 = 2vu \\ c_0 = v^2 + u^2 \end{cases}$$

In this formulas any pair of natural numbers is used :

(99) $$v > u$$

of different evenness .

Mathematical apparatus of my system of natural and irrational numbers looks like that. Conclusion of this mathematical apparatus is given in my publications, information about which is in the end of this letter.
See also:
http://yvsevolod-28.narod.ru/index.html

I DEMOSTRATE EFFECTIVENESS OF THIS MATHEMATICAL APPARATUS ON DEFINITE EXAMPLE .
I take this example from the Internet, from your publication:
Prize offered for solving number conundrum
cached/more results from this site …

In this publication you give the following example:

(100) $$3^6 + 18^3 = 3^8$$

In which the role of common multiplier make number .
Example is equivalent to my example 2, having been described above.

In this example a common multiplier is not only number 3 but also a number 3^6 with an exponent of degree n = 6 .

LET'S CONTINUE RESEARCH OF THE SYSTEM (1) AND EQUATION (92) AT THIS EXPONENT OF DEGREE.

For that we extract an equation from the system:

(101) $$a^6 + b^6 = c^6$$

Using formulas (98) and making according to condition (99),
Primary pair of numbers, composing view (83):

(102) $$\left| \begin{array}{l} v = 2 \\ u = 1 \end{array} \right.$$

We'll calculate PRIMITIVE (fundamental)
Whole-numbered solution (98) of equation (101),
that is a part of system (46):

(103) $$\left| \begin{array}{l} a_0 = v^2 - u^2 = 3 \\ b_0 = 2vu = 4 \\ c_0 = v^2 + u^2 = 5 \end{array} \right.$$

We see that the role of this solution takes first primitive Pithagorus triad .
Analyzing mathematical models (94) – (99), it is easy to see,that whole numbered solution (102) , (103) and forms (82) , (83) is invariant of all equations (95) and (92) at any exponent of degree $n \geq 2$

This is a fundamental decision of primary level of numbers, forming the system (79), including equation (101) .
Let's come to the secondary level of numbers, that make whole-numbered solutions of the system (79),having grown from the primitive triad of natural numbers , see also Yarosh Theorem and it's proof,that is given above .
Research of this phenomena of numbers theory is given on the definite equation (101), extracted from the system (79).

Using formulas (97) – (99), we calculate a common multiplier at

$$\mathbf{n = 6} :$$

(104
$$\mathbf{D_6 = (3^4 + 4^4 + 5^4)/3 = 320{,}6666667}$$

And solution of basic equation (95) with the same exponent of degree n :

(105)
$$\mathbf{a_* = \sqrt[6]{3^2 \times 320{,}6666667} = 3{,}773254116}$$

$$\mathbf{b_* = \sqrt[6]{4^2 \times 320{,}6666667} = 4{,}153003528}$$

$$\mathbf{c_* = \sqrt[6]{5^2 \times 320{,}6666667} = 4{,}473687434}$$

Using primary forms (94), we create infinite variety non-whole-numbered solutions of the equation (101).

FOR EACH EXPONENT OF DEGREE n EXISTS ITS OWN INFINITY OF VARIETY FOR SIMILAR DECISIONS.

Going backwards we will always come to primary condition (102), information about which is in view (55) of the Yarosh Theorem proof and in fundamental sum (59), containing in the basis of Principle of general (geometrical) co-variability of Pythagoras.
Hoping for your approval I give you additional information.
For the question stated by Mr. Andrew Beal «The mystery remains: is there an elementary proof?»

I have already answered and answer again:
Elementary solution of Fermat`s Last Theorem exists.
Such solution (analytical and geometrical) was published in 1993

in Moscow by «ENGINEER» publishing house as a book under title «DENOUEMENT OF MULTICENTURY ENIGMA OF DIOPHANT AND FEMAT» (The Great Fermat Theoren is finally proved for all n>2) , [8] , [9]. Formulas (94) – (97) are taken from that book.

One copy is available in library of USA Congress.

In the same 1993 above-mentioned elementary proof was published in collection of scientific works «Algorithms and structures of data development systems » by Tula State Technical University, [10].

Also in 1995 , in collection of scientific works under the same title In Tula State University was published my article under title «About some faulty statement in theory of numbers», in which it was proved that statement : «If theorem is proved for n=4, there is no need to prove it for all even exponents of degree for Fermat equation» is faulty.

More information about my proof you can find on my sites in the Internet :

http://yvsevolod-26.narod.ru//index.html ;

http;/yvsevolod-28.narod.ru/index.html ;

The value of my elementary proof is that they end with calculated formulas, supposed for calculation of infinite variety of solution of the Last theorem by P.Fermat ,theorems – hypothesis by A. Beal and theorem by V. Yarosh as a common phenomena of numbers theory.

This characteristic of my proof makes it absolutely different from publication «Wiles A. 1995. Modular elliptic curves and Fermat*s Last Theorem. Annals of Mathematics 141:443.» , See [4] .

Analyzing whole mathematical apparatus described in my letter, it's hard not to agree with Leopold Cronecker and Pythagoras.

Cronecker said:

«God created whole numbers. The others created by human»

Pythagoras knew structure of the Universe:

«The beginning of everything is one.. One as an account owns indefinite double. From one and indefinite double come numbers. From numbers come points. From points come lines. Flat figures come from them. From flat figures come volumetric figures. From them- sensed bodies»

The last statement is really astonishing. As all interacting particles - andrones- - are ruled by the low of conservation of quantum figures:

$$1 + 2 = 3$$

Description of this physical phenomena you can find on pages of my site :

http://yvsevolod-26.narod.ru/index.html

in reference: «3m Phenomenon (Formerly unknown feature of collective behavior of elementary particles)».

Finally I'd like to mention the following phenomenon of human mind.

A.Beal . as P.Fermat, is gifted with a nature of mathematical intuition, which helped him to formulate a unique rather soluble problem of figures theory without any mathematical constructions.

CONSEQUENCE

We have solution, look (79) – (86), equations :

(a) $$A^x + B^y = C^z$$

If

(b) $$A^p x + B^q y = C^r z$$

then we have solution of equations :

© $$A^p + B^q = C^r$$

as solution equations (a), if one takes into account that :

$$x = 1$$
$$y = 1$$
$$z = 1$$

For variable (x, y, z) **we have equations (b) .**

If

$$x = \text{INTEGER } K_x$$
$$y = \text{INTEGER } K_y$$

then we have equations (b), as equations :

(d) $$A^p K_x + B^q K_y = C^r z$$

From thes equations we derive the following formula for determining z **:**

(e) $$z = (A^P K_x + B^q K_Y) / C^r$$

Bring to conformity :

$$x = (C^r K_z - B^q K_y) / A^P$$
$$y = \text{INTEGER } K_y$$
$$z = \text{INTEGER } K_Z$$

$$z = \text{INTEGER } K_z$$

and

$$y = (C^r K_z - A^p K_x) / B^q$$

$$x = \text{INTEGER } K_x$$

PART № 5

G E N E R A L   C O N S E Q U E N C E

It is 9 TYPS of positive whole numbers :

$$\mathbf{N},\ (\mathbf{v}>\mathbf{u}),\ (\mathbf{a}_0,\mathbf{b}_0,\mathbf{c}_0),$$
$$,(\mathbf{p},\mathbf{q},\mathbf{r}),\ \mathbf{n},\ (3\mathbf{F'}_a, 3\mathbf{F'}_b, 3\mathbf{F'}_c),\ (\mathbf{A},\mathbf{B},\mathbf{C}),$$
$$(\mathbf{x},\mathbf{y},\mathbf{z})\ (\mathbf{X}^*,\mathbf{A}^*,\mathbf{B}^*)$$

and following forms:

$$(a'_1\alpha'_1 + a'_2\alpha'_2 + a'_3\alpha'_3)/3F'_a = 1$$
$$(b'_1\beta'_1 + b'_2\beta'_2 + b'_3\beta'_3)/3F'_b = 1$$
$$(c'_1\gamma'_1 + c'_2\gamma'_2 + c'_3\gamma'_3)/3F'_c = 1$$

$$a = \sqrt[n]{F'_a}$$
$$b = \sqrt[n]{F'_b}$$
$$c = \sqrt[n]{F'_c}$$

$$F'_a = a'_1 \times \alpha'_1 = a'_2 \times \alpha'_2 = a'_3 \times \alpha'_3$$
$$F'_c = c'_1 \times \gamma'_1 = c'_2 \times \gamma'_2 = c'_3 \times \gamma'_3$$

as equivalent of 9 secondary reduction forms of theory numbers by H.Poinkare :

$$S_1 = \begin{vmatrix} ha_1....ha_2....ha_3 \\ kb_1....kb_2....kb_3 \\ lc_1....lc_2....lc_3 \end{vmatrix}$$

$$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 1$$
$$a_1 = \beta_2\gamma_3 - \beta_3\gamma_2$$
$$a_2 = \beta_3\gamma_1 - \beta_1\gamma_3$$
$$a_3 = \beta_1\gamma_2 - \beta_2\gamma_1$$

Here (a, b, c) is endless series calculate roots of equation by P.Fermat :

$$a^n + b^n = c^n$$

end

$$(X^*, A^*, B^*)$$

whole numbers for equations of non-modular elliptic curves :

$$Y^2 = (X^* - A^*) \times X^* \times (X^* + B^*)$$

Here numbers A^*
WILL NOT DIVIDE INTO

$$16 = 4 \times (b_0 = 2vu) = 4 \times (b_0 = 2 \times 2 \times 1)$$

and

(A, B, C), whole numbers for system equations of A.Beal :

$$A^x + B^y = C^z$$
$$A^p x + B^q y = C^r z$$

WHISH HAVE MULTITUDE COMMON WHOLE MULTIPLYERS

Comment

According «Journal de I'Ecole Polytechnique,1882, Cachier 51, 45-91, part 12» :

S_1 is matrix of H.Poincare :

$$S_1 = \begin{vmatrix} a_1h & a_2h & a_3h \\ b_1k & b_2k & b_3k \\ c_1l & c_2l & c_3l \end{vmatrix}$$

If

$$h = a_0^{\,2}\ ,\ k = b_0^{\,2}\ ,\ l = c_0^{\,2}$$

then we have matrix :

$$S^*{}_1 = \begin{vmatrix} a_1 a_0^{\,2} & a_2 a_0^{\,2} & a_3 a_0^{\,2} \\ b_1 b_0^{\,2} & b_2 b_0^{\,2} & b_3 b_0^{\,2} \\ c_1 c_0^{\,2} & c_2 c_o^{\,2} & c_3 c_0^{\,2} \end{vmatrix}$$

Here whole numbers

(a_1, a_2, a_3) , (b_1, b_2, b_3) **and** (c_1, c_2, c_3) **are mutually simple numbers, according of forms (71) – (73) .**

Matrix S_1
have determinant :

$$\det S_1 = 1$$

Matrix $S^*{}_1$
have determinant :

$$\begin{aligned} \det S^*{}_1 &= a_1 (a_0^{\,2} \times b_0^{\,2} \times c_0^{\,2}) \times (c_2 b_3 - b_2 c_3) + \\ &+ a_2 (a_0^{\,2} \times b_0^{\,2} \times c_0^{\,2}) \times (c_3 b_1 - b_3 c_1) + \\ &+ a_3 (a_0^{\,2} \times b_0^{\,2} \times c_0^{\,2}) \times (c_1 b_2 - c_2 b_1) = 0 \end{aligned}$$

AS/OR

$$(c_2 b_3 - b_2 c_3) = (c_3 b_1 - b_3 c_1) = (c_1 b_2 - c_2 b_1) = 0$$

CONSEQUENTLY:

1) **According of form (37) ,**

constituent part of $\det S^*{}_1$:

$$(a_0{}^2 \times b_0{}^2 \times c_0{}^2) =$$
$$= (X^* - A^*) \times X^* \times (X^* + B^*) =$$
$$= Y^2$$

IS EQUATIONS FOR NON-MODULAR ELLIPTIC CURVES OF FIRST TYPE

2) **According $\det S^*{}_1 = 0$ and according of forms (74), (68), (71) – (73) , we have calculate system equations :**

$$(\beta'_2\,\gamma'_3 - \beta'_3\,\gamma'_2) = (c_2 b_3 - b_2 c_3) = 0$$
$$(\beta'_3\,\gamma'_1 - \beta'_1\,\gamma'_3) = (c_3 b_1 - b_3 c_1) = 0$$
$$(\beta'_1\,\gamma'_2 - \beta'_2\,\gamma'_1) = (c_1 b_2 - c_2 b_1) = 0$$

3) **At last we have come to a finale :**

According of form (39) we have :

$$(X^* - A^*) = a_0{}^2$$
$$X^* = b_0{}^2$$
$$(X^* + B^*) = c_0{}^2$$

as calculate solutions of equations for non-modular elliptic curves :

$$Y^2 = (X^* - A^*) \times X^* \times (X^* + B^*) =$$
$$= (a_0{}^2 \times b_0{}^2 \times c_0{}^2)$$

Here numbers A^* will not divide into/by

$$16 = 4 \times (b_0 = 2vu) = 4 \times (b_0 = 2 \times 2 \times 1)$$

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P O S T S C R I P T U M

For reservation of hugh reputation of mathematical school of France and mathematical school of Russia, from my point of view, it is advisable to publish in academic press a description of the following facts:

1. Secondary forms of Numbers Theory by H. Poincare include the definite algorithm of the proof of the last theorem by P. Fermat, see. [1] , [2] , [3].

2. In 1993, in Russia was published a book, [3] , in Russian and English languages. In this book the algorithm of geometrical proof of the Last theorem is described. Algorithm is based on 9 invariant triplets given in the book under numbers (1.6) – ((1.14). Those triplets are elements of secondary forms by H. Poincare. Completeness of my proof is characterized by the fact, that it (proof) is finished with formulas , see [3] , page 7 , for calculation of all roots for Fermat's equation :

$$a_*^n + b_*^n = c_*^n$$

at all even and odd indicators of degree n .

Unfortunately, in 1993 I was not acquainted with theory of numbers by H. Poincare and for that reason I didn't refer to his works. Mrs. M. Flay has that book. I sent it by post in 08.07.04 .

3. Hypothesis by Shimura-Taniyama is wrong and proof of A.Wills is questionable because there is a great variety of non-modular elliptic curves information about which is in equations by G.Frey , [4]:

(1)
$$Y^2 = (X - a^n) \times X \times (X + b^n)$$
$$Y^2 = (X - A) \times X \times (X + B)$$

This fact is easily illustrated wuth the help of equation of my elliptical curve:

(2)
$$Y^2 = a_0^n \times b_0^n \times c_0^n$$

which comes from equation of G.Frey at following substitutions:

(3)
$$(X - A) = a_0^n$$
$$X = b_0^n$$
$$(X + B) = c_0^n$$

Here:

(4)
$$a_0 = v^2 - u^2$$
$$b_0 = 2vu$$
$$c_0 = v^2 + u^2$$

primitive triads by Pifagora and $v > u$ are natural numbers of different eventy.

And:

(5)
$$n = 2 \text{ or } n > 2$$

It is known, that Frey's curve demonstrates features which are deeply different from feature, see. Chapter X1.2 , paragraph A in the book [4]. I used this difference constructing my elliptical curve, see. (2). Unlike A.Willis, my method of proof is DEDUCTIVE.

I construct ready forms of decisions being led by INTUITION.
Virtue of this method is very well described in book by R.Courant and H.Robbins "What is Mathematics?" , see beginning of the book [5].

Let's envisage properties of my curve .
Let's figure out MINIMAL DISCRIMINANT of the curve, see. [4], for first primitive triad:

(6)
$$\mathbf{a_0 = (2^2 - 1^2) = 3}$$
$$\mathbf{b_0 = (2 \times 2 \times 1) = 4}$$
$$\mathbf{c_0 = (2^2 + 1^2) = 5}$$

at minimal n=2 :

(7)
$$\mathbf{\Delta = [(a_0 \times b_0 \times c_0)^{2n}] / 2^8 = 50625}$$

For simple n =5, minimal discriminant is equal to:

(8)
$$\mathbf{\Delta = 23.6196 \times 10^{14}}$$

As far as discriminants are not equal to zero, curves are NON-SINGULAR. So those are ELLIPTICAL CURVES.

To this fact also refers the fact that simple n=2 DOES NOT DEVIDE its discriminant (7).

Experts know, why number 16 has a meaning of "litmus paper" in theory of elliptical curves.
Without details let's demonstrate this feature of number 16 on definite example for primitive triad (6).
At n=5 my curve gets determined expression:

(9)
$$\mathbf{Y^2 = 243 \times 1024 \times 3125 = 777600000}$$

At that:
16 devides 243 with oddment 3
16 devides 1024 with oddment 0
16 devides 3125 with oddment 5
16 divides number:

$$\mathbf{A = (b_0{}^5 - a_0{}^5) = 1024^5 - 243^5 = 1125.0526 \times 10^{12}}$$

with oddment, approximate, 5 .

It means that numbers forming the given elliptical curve can't be compared by module d = 16 .

CONCLUSION :

ELLIPTICAL CURVE IS NON–MODULAR
HYPOTHESIS BY SHIMURA –TANIYAMA IS WRONG
PROOF OF A.WILLIS IS UNCERTAIN

I attach full description of the mentioned problem in form of TWO specialized articles : [2] and [6] , which can be considered as MAIN ONES in complicated number of articles having been sent to the address of Mrs. M.Flay before.

Numbers theory by H.Poincare has a geometrical interpretaton, see. [1]. Equation by P.Fermat :

(10) $$a_0^n + b_0^n = c_0^n$$

also has geometrical interpretaytion ,see. [2] , at pages 20-25, 35 and 41. Equation (2) also has geometrical interpretation.

According (4) , to every pair of numbers $v > u$ of different evenness , in double dimensional space corresponds a point. To each triad of primitive Pythagora triads corresponds a point in 3-dimensional space. To each quaternion of numbers $(a_0; b_0; c_0; n)$ corresponds a point in 4-dimensional space. Infinite number of regulated ensembles of such points makes infinite number of my non-modular elliptical curves.

As example – system equations:

$$\begin{cases} Y^2 = (X - A) \times X \times (X + B) \\ a_0^n + b_0^n = c_0^n \end{cases}$$

IF:

$$\begin{cases} (X - A) = a_0^n \\ X = b_0^n \\ (X + B) = c_0^n \end{cases}$$

THEN:

$$\begin{cases} A = (b_0^n - a_0^n) \\ B = a_0^n \end{cases}$$

ACCORDING:

$$Y^2 = (b_0^n - b_0^n + a_0^n) \times b_0^n \times (b_0^n + a_0^n) =$$
$$= a_0^n \times b_0^n \times c_0^n$$

OR EQUIVALENT :

$$Y^2 = (a_0^n \times b_0^{2n}) + (a_0^{2n} \times b_0^n)$$

Equations (1), (2) and (10) have genetic links (direct and reverse) with secondary given forms of numbers theory by H.Poincare and solution of the problem of invariants by Gordan, which was made by David Gilbert , see. [3] at pages 6 and 30 . Some of this links have their own graphical representation, see. [3] at page 9.

All my proofs are finished with formulas for calculation of what is being proved.

For example at page 7 of the book [3] are given formulas for calculation of roots of P.Fermat's equation for all indicators of degree n even or odd.

This appeal I'm sending to you by ordinary mail and e mail.
Two mentioned articles I'm sending you only by e-mail because of the reason of my financial inability. I am retired, I'm 80 years old and my pension is not big.

Veuillez agreer l'expression de mes sentiments distingues.

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