PART № 3

SECONDARY REDUCTION FORMS OF H.POINCARE'S THEORY OF NUMBERS EXACTLY SOLVE FERMAT'S EQUATION AT ALL $n \geq 2$

In the first publication [5] of theory of numbers, in the introduction,
Poincare marks the following:
«Arithmetic research of homogenous forms is one of the most interesting questions of theory of numbers and of the questions , which are most interesting for geometrysts.» See. [5].
This statement of Poincare is enough realized, but not formulated yet, Principle of universal co-variation . See. [2] .

The aim of this article is to confirm geometric essence of one special paragraph of Poincare's theory of numbers which contains information about exact geometric proof of The Last theorem by Fermat.

For that purpose let's apply to publication [6] by Poincare,
translated into Russia , see. [7]
Here I give a quotation [7] :

«Everything we have spoken before, can be applied only to main reduction forms, so to them
we can give the following results:
1) Each class commonly saying is only one main reduction form;
2) There are infinitely many classes;
3) Main reduction forms are divided into three types;
4) Form of the first and second type is a finite number;
5) Forms of the third type are divided into infinite multitude of sorts, and each sort contains infinitely mane reduction forms.
Let's attend to secondary reduction forms.»

From analysis of these forms let's choose a fragment which has a link to geometric proof of
The Last theorem by Fermat see. [8] and [9] .
This fragment Poincare explains the following way, see p.889 in [6] :

«As three simple numbers a_1, a_2, a_3 are mutually simple, there are always
Nine Whole Numbers, satisfying the following conditions:

$$
(46)\qquad \left|\begin{array}{l}
a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 1 \\
a_1 = \beta_2\gamma_3 - \beta_3\gamma_2 \\
a_2 = \beta_3\gamma_1 - \beta_1\gamma_3 \\
a_3 = \beta_1\gamma_2 - \beta_2\gamma_1
\end{array}\right.
$$

Then instead of substitutions, used by Poincare we will use substitutions by Frei , see. [3] , which uses Frei at research of features of his elliptical curve:

(47) $$Y^2 = (X - A)X(X + B)$$

As in Poincare's, here are used mutually simple numbers.
Then we get convinced that substitutions by Frei in equation (47) correspond to one of the reduction forms, number of which in each sort of forms of the Third type is infinite. We will get convinced that my substitutions see (50), widening substitutions of Frei,
brings researcher to geometrical proof of the Last theorem by Fermat (LTF), which reader can find in [8] and [9].

Frei uses the following substitutions:

(48) $$A = a^q \qquad \text{and} \qquad B = b^q$$

As far as Fermat's equation:

(49) $$a^n + b^n = c^n$$

contains three members, the author [8] forms not two, but three substitutions, changing simple number q for any whole number $n \geq 2$:

(50) $$\left| \begin{array}{l} A = a^n \\ B = b^n \\ C = c^n \end{array} \right.$$

In the result Fernat's equation gets simple phenomenologic formulas for calculation of its PRIMITIVE SOLUTIONS at any index of degree $n \geq 2$:

(51) $$\left| \begin{array}{l} a_* = \sqrt[n]{A} \\ b_* = \sqrt[n]{B} \\ c_* = \sqrt[n]{C} \end{array} \right.$$

Any non-primitive solutions of Fermat's equation are calculated by simple multiplying of primitive solutions to any common multiplier S :

(52) $$\left| \begin{array}{l} a = a_* \times S \\ b = b_* \times S \\ c = c_* \times S \end{array} \right.$$

Further widening of infinite number of calculated solutions of Fermat's equation are done with the help of any root multiplier, including special multiplier:

(53) $$D_n = (a_0^{\ n-2} + b_0^{\ n-2} + c_0^{\ n-2})/3$$

In this case we get universal forms for solution of Fermat's equation:

(54) $$\left| \begin{array}{l} a = \sqrt[n]{A \times D_n} \times S \\ b = \sqrt[n]{B \times D_n} \times S \\ c = \sqrt[n]{C \times D_n} \times S \end{array} \right.$$

Construction of calculated solutions of equation by Fermat Is finished with intuitive construction of rtiad of mutually simple numbers A, B, C :

(55) $$\left| \begin{array}{l} A = a_0^{\ 2} \\ B = b_0^{\ 2} \\ C = c_0^{\ 2} \end{array} \right.$$

Here as in construction of multiplier D_n, are used primitive Pythagora's triads:

(56) $$\left| \begin{array}{l} a_0 = v^2 - u^2 \\ b_0 = 2vu \\ c_0 = v^2 + u^2 \end{array} \right.$$

being constructed from any pair v > u of natural numbers of different evenness.

To tie up take by us calculated solutions of the equation by Fermat with substitutions of Poincare , let's remember mentioned above remark of Poincare that arithmetical theory of numbers has geometrical interpretation .

Described above formula for calculation of roots for Fermat's equation also have geometrical interpretation .
This interpretation is described in [8] and [9].

The interpretation is based on building of nine triads of rectangles - squares of Diophant. Every three triads of squares make indivisible geometrical variety consisting of three isometric triangles of Diophant.

Below I give an illustration of the above said algorithm with the help of Fig.10 , taken from [8] and [9] :

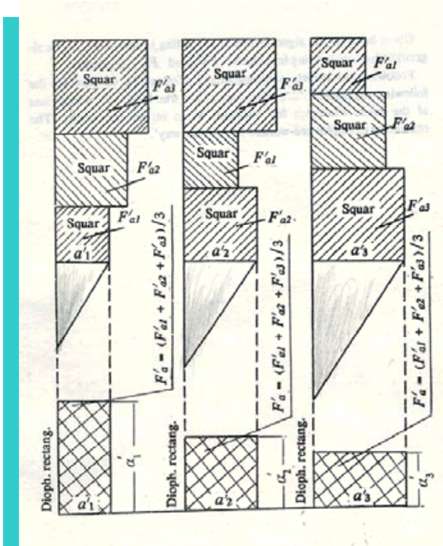

Fig. 10. Three variants of construction of equidimensional in area Diophantine rectangles, yielding the same result — smaller invariant F'_a of Diophantine space

Fig.10

On this picture is given a construction of three triads of triangles - squares of Diophant, which comes to construction of three isometric areas $\mathbf{F'_a}$ of triangles of Diophant. The area $\mathbf{F'_a}$ is the smallest invariant of Diophant, defining the smallest root for Fermat's equation.

The same way are made medium and biggest invariants which define medium and biggest roots for Fermat's equation:

(57)
$$\left| \begin{array}{l} \mathbf{F'_a = a_0{}^2 \times D_n} \\ \mathbf{F'_b = b_0{}^2 \times D_n} \\ \mathbf{F'_c = c_0{}^2 \times D_n} \end{array} \right.$$

Below, see Fig.10, we see three isometric by area $\mathbf{F'_a}$ triangles of Diophant. Above, on the cathetus of corresponding rectangular triangles are made three triads of own rectangles-squares of Diophant, medium arithmetic meaning of the areas of which is equak to three isometric areas of the corresponding rectangles of Diophant, drawn in the bottom of Fig. 10 .

The result of such geometric constructions is a construction of three MAIN algebraic invariants , see.(57). From Fig.10 comes construction of the FIRST (smallest) invariant in which are used marks of sides of the rectangle drawn in the bottom of the drawing :

(58)
$$\mathbf{F'_a = a'_1 \times \alpha'_1 = a'_2 \times \alpha'_2 = a'_3 \times \alpha'_3}$$

The same way are made
SECOND (medium) invariant:

$$F'_b = b'_1 \times \beta'_1 = b'_2 \times \beta'_2 = b'_3 \times \beta'_3 \qquad (59)$$

and THIRD (biggest) invariant :

$$F'_c = c'_1 \times \gamma'_1 = c'_2 \times \gamma'_2 = c'_3 \times \gamma'_3 \qquad (60)$$

In these geometric models the base of the rectangles of Diophant is equal to smallest cathetus of corresponding rectangular triangles areas of which are not isometric see Fig.10 .
Invariants (58) – (60) are connecting links between forms of Poincare see. (46) , and forms (50) , made according to forms of Frei (48) .
At last these invariants make the basis for formulas (54), with the help of which are calculated roots of Fermat's equation (49) .

Proof of the above said .
According to [8] and [9] in space of Diophant's variety one can build NINE invariant algebraic forms, numeric meanings of which are defined HIGHTS of Diophant's rectangles , see Fig.10:

$$(61) \quad \left| \begin{array}{l} \alpha'_1 = (a_0 \times D_n) / \sqrt{a_0^{\,n-2}} \\ \alpha'_2 = (a_0 \times D_n) / \sqrt{b_0^{\,n-2}} \\ \alpha'_3 = (a_0 \times D_n) / \sqrt{c_0^{\,n-2}} \end{array} \right.$$

$$(62) \quad \left| \begin{array}{l} \beta'_1 = (b_0 \times D_n) / \sqrt{a_0^{\,n-2}} \\ \beta'_2 = (b_0 \times D_n) / \sqrt{b_0^{\,n-2}} \\ \beta'_3 = (b_0 \times D_n) / \sqrt{c_0^{\,n-2}} \end{array} \right.$$

$$(63) \quad \left| \begin{array}{l} \gamma'_1 = (c_0 \times D_n) / \sqrt{a_0^{\,n-2}} \\ \gamma'_2 = (c_0 \times D_n) / \sqrt{b_0^{\,n-2}} \\ \gamma'_3 = (c_0 \times D_n) / \sqrt{c_0^{\,n-2}} \end{array} \right.$$

At that, bases of Diophant's rectangles, see Fig.10, are SEGMENTS LENGTHS of which correspondingly are:

(64)
$$\begin{aligned} a'_1 &= a_0 \times \sqrt{a_0}^{\,n-2} \\ a'_2 &= a_0 \times \sqrt{b_0}^{\,n-2} \\ a'_3 &= a_0 \times \sqrt{c_0}^{\,n-2} \end{aligned}$$

(65)
$$\begin{aligned} b'_1 &= b_0 \times \sqrt{a_0}^{\,n-2} \\ b'_2 &= b_0 \times \sqrt{b_0}^{\,n-2} \\ b'_3 &= b_0 \times \sqrt{c_0}^{\,n-2} \end{aligned}$$

(66)
$$\begin{aligned} c'_1 &= c_0 \times \sqrt{a_0}^{\,n-2} \\ c'_2 &= c_0 \times \sqrt{b_0}^{\,n-2} \\ c'_3 &= c_0 \times \sqrt{c_o}^{\,n-2} \end{aligned}$$

Let's pay attention that MAIN invariants, see. (58) – (60), are constructed from invariants (61) – (66) .

At last we have come to a final.
All the chain of described above substitutes is closed on conditions conditions of Poincare's substitutes ,see. (46) .

First condition:

(67)
$$a'_1\alpha'_1 + a'_2\alpha'_2 + a'_3\alpha'_3 = 1$$

in our case it is widened up to three corresponding conditions:

(68)
$$\begin{aligned} &(a'_1\alpha'_1 + a'_2\,\alpha'_2 + a'_3\alpha'_3)/3F'_a = 1 \\ &(b'_1\beta'_1 + b'_2\beta'_2 + b'_3\beta'_3)/3F'_b = 1 \\ &(c'_1\gamma'_1 + c'_2\gamma'_2 + c'_3\gamma'_3)/3F'_c = 1 \end{aligned}$$

At that, formation of Poincare's conditions, given in the beginning of the article :

«As three whole numbers a_1, a_2, a_3 are mutually simple ,
there are always NINE whole numbers ,
satisfying the following conditions :

(69)
$$\begin{vmatrix} a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 1 \\ a_1 = \beta_2\gamma_3 - \beta_3\gamma_2 \\ a_2 = \beta_3\gamma_1 - \beta_1\gamma_3 \\ a_3 = \beta_1\gamma_2 - \beta_2\gamma_1 \end{vmatrix}$$

make sense , coordinated with geometrical proof
of the Last theorem by Fermat:
So if three whole numbers

(70)
$$a_0, b_0, c_0$$

making each triad of primitive Pythagora's numbers,
are mutually simple, it means that at all indexes of degree n ,
there are always NINE WHOLE NUMBERS:

(71)
$$\begin{vmatrix} a_1 = a_1'^2 = a_0^2(a_0^{n-2}) \\ a_2 = a_2'^2 = a_0^2(b_0^{n-2}) \\ a_3 = a_3'^2 = a_0^2(c_0^{n-2}) \end{vmatrix}$$

(72)
$$\begin{vmatrix} b_1 = b_1'^2 = b_0^2(a_0^{n-2}) \\ b_2 = b_2'^2 = b_0^2(b_0^{n-2}) \\ b_3 = b_3'^2 = b_0^2(c_0^{n-2}) \end{vmatrix}$$

(73)
$$\begin{vmatrix} c_1 = c_1'^2 = c_0^2(a_0^{n-2}) \\ c_2 = c_2'^2 = c_0^2(b_0^{n-2}) \\ c_3 = c_3'^2 = c_0^2(c_0^{n-2}) \end{vmatrix}$$

that satisfy three conditions (68).

At that three other conditions of Poincare turn into zero :

(74)
$$\left|\begin{array}{l} \beta'_2\gamma'_3 - \beta'_3\gamma'_2 = 0 \\ \beta'_3\gamma'_1 - \beta'_1\gamma'_3 = 0 \\ \beta'_1\gamma'_2 - \beta'_2\gamma'_1 = 0 \end{array}\right.$$

As a result Fermat's equation, see (46), gets easily calculated formula, calculation of which comes to calculation of areas of rectangles of Diophant, that means calculation of MAIN invariants (58) – (60) .

In this case roots of Fermat's equation (46) are calculated with the help of the following formulas:

(75)
$$\left|\begin{array}{l} a = \sqrt[n]{F'_a} \times S \\ b = \sqrt[n]{F'_b} \times S \\ c = \sqrt[n]{F'_c} \times S \end{array}\right.$$

(76)
$$\left|\begin{array}{l} a = a_* \times S \\ b = b_* \times S \\ c = c_* \times S \end{array}\right.$$

These formulas as it was shown above, have direct and reverse connection with theory of numbers of H.Poincare and geometrical variety of Diophant and Pythagora.

All said above corresponds to Principle of universal (geometrical) co-variation, which is a basis of all fundamental research of the twentieth century, see. [2] , and simple geometric proofs of the Last theorem of Fermat, which don't need any use of elliptic curves and modular forms .

If reader gets acquainted with web-sites
http://yvsevolod-26.narod.ru/index.html
http://int20730601.narod.ru/index.html
http://yvsevolod-29.narod.ru/index.html
http://yvsevolod-28.narod.ru/index.html
kept in the Narod catalogues of Russian Internet,
he will get convinced in simple but easily proved truth:

Harmony of space, harmony of life on the Earth and Universe are reflected in great harmony of natural numbers so capaciously and variously described in Theory of Numbers by H.Poincare.

Note: Correctness of given here substitutes connecting one of the secondary given forms of Poincare with geometric proof of LTF ,can be checked by easy calculations on pocket calculator making any pair (v > u)of natural numbers of different evenness and calculating with a formula (56) corresponding primitive triad of Pythagora..